Exam. Code : 103202
Subject Code : 7438

B.A./B.Sc. 2nd Semester (Old Syllabus of 2015)

MATHEMATICS

Paper—II

(Calculus—II)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

I. (a) Discuss the continuity of the function

$$f(x, y) = \begin{cases} xy \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at the point (0, 0).

(b) If $x^x y^y z^z = C$, show that $\dfrac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$, when $x = y = z$. 5;5

II. (a) If $z = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then using Euler's theorem on homogenous functions, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}z$.

(b) Show that $f(x, y) = \sin x + \cos y$ is differentiable at every point of R^2. 5,5

III. (a) Expand $e^{ax}\cos by$ in powers of x and y upto first four terms.

(b) If $x = r\cos\theta\cos\varphi$, $y = r\sin\theta\sqrt{1-m^2\sin^2\varphi}$ and $z = r\sin\varphi\sqrt{1-n^2\sin^2\theta}$, where $m^2 + n^2 = 1$ then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)} = \frac{r^2(m^2\cos^2\varphi + n^2\cos^2\theta)}{\sqrt{(1-m^2\sin^2\varphi)(1-n^2\sin^2\theta)}}.$$

5,5

IV. (a) Find the envelop of the family of lines $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$, θ being the parameter.

(b) Find the evolute of the curve :

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right),\ y = a\sin t.$$ 5,5

V. (a) In a plane triangle, find the maximum value of cos A cos B cos C.

(b) Find the maximum and minimum value of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$.

5,5

SECTION—B

VI. (a) Prove that

$$\iint_R x^{2p-1} y^{2q-1} dxdy = \frac{a^{2(p+q)} . \Gamma(q)\Gamma(p+1)}{4p.\Gamma++(p+q+1)}$$

for all positive values of x and y lying in $x^2 + y^2 \le a$.

(b) Change the order of integration of

$$\int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{2x}} f(x, y)\, dy\, dx .$$ 5,5

VII. (a) Find the area of the region bounded by $y^2 = -x$ and $y^2 = x + 2y$.

(b) Evaluate $\iint_A (x^2 + y^2)\, dxdy$, where A is the region bounded by the four hyperbolas $x^2 - y^2 = 2, 9$ and $xy = 2, 4$. 5,5

VIII. (a) Find the volume of the cylinder with base radius r and height h.

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 \frac{dzdydx}{\sqrt{x^2+y^2+z^2}}$. 5,5

IX. (a) Evaluate $\iiint (x + y + z + 1)^2 \, dx \, dy \, dz$ throughout the region defined by :

$$x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1.$$

(b) If $(\bar{x}, \bar{y}, \bar{z})$ be the centre of gravity of the volume cut off from the cylinder $z^2 + y^2 = 2az$ by the planes $x = mz$ and $x = nz$ find $\bar{z}$. 5,5

X. (a) Find the moment of inertia of a homogenous solid bounded by the cylinder $x^2 + y^2 = a^2$ and the planes $z = 0$, $z = b$ about y – axis, ρ being the density.

(b) Find the moment of inertia of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$ with uniform unit mass density about z-axis. 5,5